Impact of Channel Knowledge on Cognitive Radio System Capacity

Pawel A. Dmochowski[*], Himal A. Suraweera[†], Peter J. Smith[‡], Mansoor Shafi[§]

[*] School of Engineering and Computer Science, Victoria University of Wellington, Wellington, New Zealand
[†] Department of Electrical and Computer Engineering, National University of Singapore, Singapore
[‡] Department of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand
[§] Telecom New Zealand, Wellington, New Zealand
Email:pdmochowski@ieee.org, elesaha@nus.edu.sg, p.smith@elec.canterbury.ac.nz, mansoor.shafi@telecom.co.nz

Abstract—We examine the impact of channel knowledge on the secondary user (SU) in a cognitive radio system. Under a minimum signal-to-interference-and-noise ratio (SINR) constraint for the primary user (PU) receiver, we determine the SU capacity under four channel knowledge scenarios. We derive analytical expressions for the capacity cumulative distribution functions which are verified by means of simulations. We show that the lack of exact knowledge of the PU-PU channel gain by the SU-Tx either prohibits SU transmission or necessitates high interference level at the PU. We also show that the lack of exact knowledge of the SU-Tx to PU-Rx link has little or no impact on SU capacity.

I. INTRODUCTION

The cognitive radio concept, introduced in [1], refers to a smart radio which can sense the external electromagnetic environment and adapt its transmission parameters according to the current state of the environment. Secondary (or Cognitive) users (SUs) can be designed to access parts of the primary user (PU) spectrum for their information transmission, provided they cause minimal interference to the PUs in that band [2].

A large number of papers have appeared on various aspects of cognitive radio systems, including fundamental information theoretic capacity limits (see, for example, [3]–[10]). The cognitive radios can protect the PU transmissions by regulating the SU transmitter to PU receiver interference power to be below a well defined threshold level. The limits on this received interference level at the PU receiver can be imposed with an average/peak constraint [4]. Another method of protecting the PU transmission is to consider a minimum value for its signal-to-interference noise ratio (SINR) beyond which further degradation is not accepted. Note that if the PU signal has a signal-to-noise ratio (SNR) below this level then the SU cannot transmit at all since the minimum SINR is unobtainable. Clearly with the SINR constraint, depending on the fading level of the PU-Tx to PU-Rx link, the conservatism inherent in the constant interference threshold constraint can be relaxed to some extent. Hence, with an SINR constraint, the tolerable PU interference is no longer a constant and this can be to the benefit of the SU-Tx when the PU link is strong. The price of this relaxation is that some information about the PU-Tx to PU-Rx link must be available to the SU-Tx.

The motivation for our work arises from the fact that complete channel knowledge acquisition of all links in the network is impractical. Generally in wireless systems, various
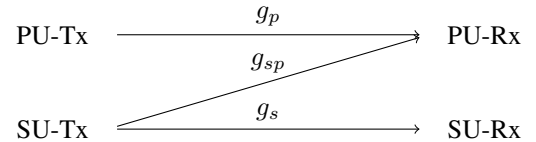

Fig. 1. System Model

levels of channel knowledge (either in the form of instantaneous or statistical channel information) on radio links are collected for system optimization. Moreover, due to estimation errors, the acquired channel information is never perfect. Therefore, an important question is the impact of the nature of channel knowledge availability on the cognitive radio capacity. Previously, [6] derived SU capacity under a peak-power constraint and asymmetric channel conditions, while [7] examined the impact of instantaneous channel estimation errors on the SU capacity. Note that [7] only considered imperfect channel state information (CSI) for the SU-PU link. In this paper we consider the effect of statistical CSI rather than instantaneous channel estimation errors. Furthermore, the investigation considers the effect of statistical CSI on all the channels for the primary and secondary links under both SINR and peak power constraints.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a cognitive radio system where a SU link consisting of a SU-Tx and a SU-Rx transmits simultaneously over the same narrow band with a PU link consisting of a PU-Tx and a PU-Rx. Independent point-to-point flat Rayleigh fading channels are assumed for all links in the network. Let $g_p = |h_p|^2$, $g_s = |h_s|^2$ and $g_{sp} = |h_{sp}|^2$ denote the instantaneous channel gains of the PU-Tx to PU-Rx, SU-Tx to SU-Rx and SU-Tx to PU-Rx links, respectively. Furthermore, we denote the exponentially distributed probability density functions (pdfs) of the random variables (RVs) g_p, g_s and g_{sp} by $f_{g_p}(x), f_{g_s}(x)$, and $f_{g_{sp}}(x)$, respectively. These distributions are governed by their corresponding parameters $\Omega_p = \mathbb{E}(g_p), \Omega_s = \mathbb{E}(g_s)$ and $\Omega_{sp} = \mathbb{E}(g_{sp})$, where $\mathbb{E}(\cdot)$ denotes the expectation operator. In this paper, depending on the type of channel knowledge available at the SU-Tx, we identify four different scenarios of interest. In all cases, it is

assumed that the SU-Tx has exact knowledge of g_s.[1]

Scenario 1: The PU-Tx to PU-Rx channel g_p and SU-Tx to the PU-Rx channel g_{sp} are perfectly known.

Scenario 2: The PU-Tx to PU-Rx channel g_p is perfectly known while only the mean Ω_{sp} of the channel between the SU-Tx and the PU-Rx is known.

Scenario 3: The mean Ω_p and the exact channel gain g_{sp} are known.

Scenario 4: Only the means Ω_p and Ω_{sp} are known.

In all cases, where possible, we impose a constraint, γ_T, on the SINR at the PU receiver, denoted by γ_p. Hence,

$$\gamma_p = \frac{P_p g_p}{P_s g_{sp} + \sigma_p^2}, \text{ and } \gamma_p \geq \gamma_T, \quad (1)$$

where γ_T is a pre-defined SINR threshold, P_p and P_s are the primary and secondary transmit powers, respectively, and σ_p^2 is the additive white Gaussian noise (AWGN) variance at the PU-Rx. Note that in the event that the PU-Rx SNR $P_p g_p / \sigma_p^2 < \gamma_T$, the SINR constraint in (1) cannot be satisfied, and thus the SU transmit power is zero. We also impose a maximum SU transmit power constraint, denoted by P_m, which arises in practice, for example, due to power amplifier nonlinearities. Thus, in *Scenarios* 1 and 2, where the SU-Tx knows g_p, the SU transmit power is given by

$$P_t = \begin{cases} 0 & \frac{P_p g_p}{\gamma_T} < \sigma_p^2 \\ \min(P_s, P_m) & \text{otherwise} \end{cases}, \quad (2)$$

where P_s is obtained from (1) by solving $\gamma_T = \gamma_p$. As will be shown in Section III, the $P_t = 0$ condition for *Scenarios* 3 and 4 will depend on the choice of system parameters and thus (2) will be modified accordingly.

Furthermore, we note that the constraints described above can only be guaranteed if the SU-Tx has perfect knowledge of the links g_p and g_{sp}, that is under *Scenario* 1. In analysing *Scenarios* 2-4, we use probabilistic contraints. Hence, we require the SINR constraint to hold with an acceptably high probability, $1 - \alpha$, where α is small.

The signal-to-noise ratio at the SU-Rx is $\frac{P_t g_s}{\sigma_s^2}$, where σ_s^2 is the AWGN variance at the SU-Rx. In our investigation of the SU capacity C, we first consider the RV $\gamma = P_t g_s$, with the pdf and cumulative distribution function (cdf) denoted by $f_\gamma(x)$ and $F_\gamma(x)$, respectively. The SU capacity is thus given by

$$C = \log_2\left(1 + \frac{\gamma}{\sigma_s^2}\right), \quad (3)$$

which allows us to obtain the cdf of C from $F_\gamma(x)$ by

$$F_C(y) = F_\gamma\left(\sigma_s^2(2^y - 1)\right). \quad (4)$$

[1]The impact of the interference caused by the PU-Tx to SU-Rx link on the SU capacity is not taken into account due to space constraints. Analysis involving this factor is, however, considered in the forthcoming full journal version of this work.

Furthermore, the mean capacity, $\bar{C}$, can be derived using $f_\gamma(x)$ by

$$\bar{C} = \mathbb{E}(C) = \int_0^\infty \log_2\left(1 + \frac{x}{\sigma_s^2}\right) f_\gamma(x) \, dx. \quad (5)$$

In what follows we derive analytical expressions for $F_\gamma(x)$ and $f_\gamma(x)$ which, using (4) and (5), allow us to compute the capacity cdf and mean capacity for each scenario.

III. SU CAPACITY

A. Scenario 1

Assuming that the SU has full knowledge of g_p, g_s and g_{sp}, P_s can be obtained directly from (1), giving

$$P_s = \frac{\frac{P_p g_p}{\gamma_T} - \sigma_p^2}{g_{sp}}. \quad (6)$$

We note that while we ignore the $P_t = 0$ case in (2), the following derivation is valid since $\Pr(\gamma > 0) = 0$ for $P_t \leq 0$. In finding $F_\gamma(x)$ we solve for the complementary cdf of γ

$$\Pr(\gamma > x) = \Pr(g_s \min(P_m, P_s) > x) \quad (7)$$

$$= \Pr\left(P_m g_s > x, \left(\frac{P_p g_p}{\gamma_T} - \sigma_p^2\right)\frac{g_s}{g_{sp}} > x\right)$$

$$= \Pr\left(g_s > \frac{x}{P_m}, \left(\frac{P_p g_p}{\gamma_T} - \sigma_p^2\right)g_s > x g_{sp}\right),$$

where $\Pr(\cdot)$ denotes probability. Noting that g_p is an exponentially distributed RV, we can rewrite (7) as

$$\Pr(\gamma > x) = \int_0^\infty \int_{\frac{x}{P_m}}^\infty e^{-\frac{\gamma_T}{P_p \Omega_p}\left(\frac{xv}{u} + \sigma_p^2\right)} f_{g_s}(u) f_{g_{sp}}(v) \, du \, dv. \quad (8)$$

Substituting for $f_{g_s}(u)$ and $f_{g_{sp}}(v)$, and changing the order of integration one obtains

$$\Pr(\gamma > x) \quad (9)$$

$$= \frac{e^{-\frac{\gamma_T \sigma_p^2}{P_p \Omega_p}}}{\Omega_{sp}\Omega_s} \int_{\frac{x}{P_m}}^\infty e^{-\frac{u}{\Omega_s}} \int_0^\infty e^{-\left(\frac{\gamma_T x}{P_p \Omega_p u} + \frac{1}{\Omega_{sp}}\right)v} \, dv \, du$$

$$= \frac{e^{-\frac{\gamma_T \sigma_p^2}{P_p \Omega_p}}}{\Omega_{sp}\Omega_s} \int_{\frac{x}{P_m}}^\infty \frac{e^{-\frac{u}{\Omega_s}}}{\frac{\gamma_T x}{P_p \Omega_p u} + \frac{1}{\Omega_{sp}}} \, du.$$

After simplifying (9), the cdf $F_\gamma(x) = 1 - \Pr(\gamma > x)$ can be shown to be [11, Equation (3.351.2)]

$$F_\gamma(x) = 1 - e^{-\frac{\gamma_T \sigma_p^2}{P_p \Omega_p}}\left[e^{-\frac{x}{P_m \Omega_s}} - \frac{\Omega_{sp}\gamma_T x}{P_p \Omega_p \Omega_s} e^{\frac{\Omega_{sp}\gamma_T x}{P_p \Omega_p \Omega_s}}\right. \quad (10)$$

$$\left. \times \Gamma\left(0, \frac{\Omega_{sp}\gamma_T x}{P_p \Omega_p \Omega_s} + \frac{x}{P_m \Omega_s}\right)\right],$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function. Finally, the capacity cdf is obtained by substituting (10) into (4).

Furthermore, we can obtain the pdf $f_\gamma(x)$ by differentiating (10) with respect to x, which after some manipulation gives

$$f_\gamma(x) = e^{-\frac{\gamma_T \sigma_p^2}{P_p \Omega_p}} \left[\left(\frac{1}{P_m \Omega_s} - \frac{\Omega_{sp} \gamma_T}{P_p \Omega_p \Omega_s} \right) e^{-\frac{x}{P_m \Omega_s}} \right. \tag{11}$$
$$+ e^{\frac{\Omega_{sp} \gamma_T x}{P_p \Omega_p \Omega_s}} \left(\frac{(\Omega_{sp} \gamma_T)^2 x}{(P_p \Omega_p \Omega_s)^2} + \frac{\Omega_{sp} \gamma_T}{P_p \Omega_p \Omega_s} \right)$$
$$\left. \times \Gamma\left(0, \frac{\Omega_{sp} \gamma_T x}{P_p \Omega_p \Omega_s} + \frac{x}{P_m \Omega_s} \right) \right].$$

Equation (11) can be used in (5) to give a single numerical integral for the mean capacity $\bar{C}$.

B. Scenario 2

Note that in *Scenarios* 2-4, exact channel knowledge is unavailable. As a result, the SU cannot guarantee that (1) is satisfied since the value of γ_p is uncertain. Hence, we constrain the SU to satisfy (1) with an acceptably high probability, $1-\alpha$, where α is usually small.

Specifically, in *Scenario* 2, where the SU knows only the mean, Ω_{sp}, of g_{sp}, we consider the probability of satisfying the SINR constraint with a probability of $1 - \alpha$, that is

$$\Pr\left(\frac{P_p g_p}{P_s g_{sp} + \sigma_p^2} \geq \gamma_T \middle| g_p, \Omega_{sp} \right) = 1 - \alpha, \tag{12}$$

which can be rewritten as

$$\Pr\left(g_{sp} \leq \frac{P_p g_p - \gamma_T \sigma_p^2}{P_s \gamma_T} \middle| g_p, \Omega_{sp} \right) = 1 - \alpha. \tag{13}$$

From (13) we can derive the expression for the transmit power P_s, that is

$$P_s = -\frac{P_p g_p - \gamma_T \sigma_p^2}{\gamma_T \Omega_{sp} \ln(\alpha)}. \tag{14}$$

From (14) the complementary cdf of γ is derived by noting that

$$\Pr(\gamma > x) = \Pr(P_m g_s > x, P_s g_s > x) \tag{15}$$
$$= \Pr\left(g_s > \frac{x}{P_m} \middle| P_m < P_s \right) + \Pr\left(g_s > \frac{x}{P_s} \middle| P_m > P_s \right),$$

which in turn can be shown to be

$$\Pr(\gamma > x) = \int_{\psi_0}^{\psi} \Pr\left(g_s > \frac{x}{P_s} \right) f_{g_p}(y) \, dy \tag{16}$$
$$+ \int_{\psi}^{\infty} \Pr\left(g_s > \frac{x}{P_m} \right) f_{g_p}(y) \, dy,$$

where $\psi_0 = \frac{\gamma_T \sigma_p^2}{P_p}$ and $\psi = \frac{\gamma_T(\sigma_p^2 - P_m \Omega_{sp} \ln(\alpha))}{P_p}$. The lower integration limit in the first term of (16) takes into account the $P_t = 0$ condition in (2). After some manipulation, we can simplify (16) to obtain $F_\gamma(x) = 1 - \Pr(\gamma > x)$ as,

$$F_\gamma(x) = 1 - \int_{\psi_0}^{\psi} \Pr\left(g_s > \frac{x}{P_m} \right) f_{g_p}(y) \, dy \tag{17}$$
$$- \int_{\psi}^{\infty} \Pr\left(g_s > -\frac{x \gamma_T \Omega_{sp} \ln(\alpha)}{P_p y - \gamma_T \sigma_p^2} \right) f_{g_p}(y) \, dy,$$

which results in

$$F_\gamma(x) = 1 - \exp\left\{ -\frac{x}{P_m \Omega_s} - \frac{\psi}{\Omega_p} \right\} \tag{18}$$
$$- \frac{1}{\Omega_p} \int_{\psi_0}^{\psi} e^{-\frac{x \gamma_T \Omega_{sp} \ln(\alpha)}{(\gamma_T \sigma_p^2 - P_p y) \Omega_s}} e^{-\frac{y}{\Omega_p}} \, dy.$$

To the best of our knowledge, there exists no closed form solution to the integral in (18), and in Section IV it is evaluated numerically.

C. Scenario 3

In *Scenario* 3, where the SU has exact knowledge of g_{sp} and knows only the mean Ω_p, we once again satisfy the SINR constraint with a probability of $1 - \alpha$. Hence,

$$\Pr\left(\frac{P_p g_p}{P_s g_{sp} + \sigma_p^2} \geq \gamma_T \middle| \Omega_p, g_{sp} \right) = 1 - \alpha, \tag{19}$$

which gives

$$\Pr\left(g_p \geq \frac{\gamma_T(P_s g_{sp} + \sigma_p^2)}{P_p} \middle| \Omega_p, g_{sp} \right) = 1 - \alpha. \tag{20}$$

Following the same approach as for *Scenario* 2 in Section III-B, noting that g_p is an exponential RV, one can show that

$$P_s = -\left(\frac{\ln(1-\alpha) P_p \Omega_p}{\gamma_T} + \sigma_p^2 \right) \frac{1}{g_{sp}}. \tag{21}$$

Defining for notational convenience

$$Q = -\left(\frac{\ln(1-\alpha) P_p \Omega_p}{\gamma_T} + \sigma_p^2 \right), \tag{22}$$

we have a transmit power for *Scenario* 3 given by

$$P_t = \begin{cases} 0 & Q < 0 \\ \min\left(P_m, \frac{Q}{g_{sp}} \right) & Q > 0 \end{cases}. \tag{23}$$

Using (22) and (23) we note that the SU transmission will only occur for $Q > 0$, which is equivalent to

$$\alpha > 1 - e^{-\frac{\gamma_T}{P_p \Omega_p / \sigma_p^2}}. \tag{24}$$

In such a case, solving for the cdf of γ, we write

$$\Pr(\gamma > x) \tag{25}$$
$$= \Pr\left(g_s > \frac{x}{P_m} \middle| P_m < \frac{Q}{g_{sp}} \right) + \Pr\left(g_s > \frac{x}{P_s} \middle| P_m > \frac{Q}{g_{sp}} \right)$$

giving

$$\Pr(\gamma > x) = \int_0^{Q/P_m} \Pr\left(g_s > \frac{x}{P_m} \right) f_{g_{sp}}(y) \, dy \tag{26}$$
$$+ \int_{Q/P_m}^{\infty} \Pr\left(g_s > \frac{xy}{Q} \right) f_{g_{sp}}(y) \, dy.$$

Upon simplifying (26), we obtain

$$F_\gamma(x) = 1 - e^{-\frac{x}{\Omega_{sp} P_m}} \left(1 - e^{-\frac{Q}{\Omega_s P_m}} \right) \tag{27}$$
$$- \frac{1}{1 + \frac{\Omega_s x}{\Omega_{sp} Q}} e^{-\frac{Q}{\Omega_{sp} P_m} - \frac{x}{\Omega_s P_m}}.$$

Here, once again we can obtain the pdf

$$f_\gamma(x) = \frac{1 - e^{-\frac{Q}{\Omega_{sp}P_m}}}{\Omega_s P_m} e^{-\frac{x}{\Omega_s P_m}} \quad (28)$$

$$+ \frac{1}{\Omega_s P_m} \frac{e^{-\frac{Q}{\Omega_{sp}P_m} - \frac{x}{\Omega_s P_m}}}{1 + \frac{\Omega_{sp}}{\Omega_s Q}x} + \frac{\Omega_{sp}}{\Omega_s Q} \frac{e^{-\frac{Q}{\Omega_{sp}P_m} - \frac{x}{\Omega_s P_m}}}{\left(1 + \frac{\Omega_{sp}}{\Omega_s Q}x\right)^2}.$$

Using (28) and (5), the mean SU capacity can be shown to be

$$\bar{C} = \frac{1 - e^{-\frac{Q}{\Omega_{sp}P_m}}}{\ln(2)} \Gamma\left(0, \frac{\sigma_s^2}{\Omega_s P_m}\right) e^{\frac{\sigma_s^2}{\Omega_s P_m}} \quad (29)$$

$$- \frac{1}{\ln(2)\left(1 - \frac{\Omega_{sp}\sigma_s^2}{\Omega_s Q}\right)} \Gamma\left(0, \frac{Q}{\Omega_{sp}P_m}\right)$$

$$+ \frac{e^{-\frac{Q}{\Omega_{sp}P_m}}}{\ln(2)\left(1 - \frac{\Omega_{sp}\sigma_s^2}{\Omega_s Q}\right)} \Gamma\left(0, \frac{\sigma_s^2}{\Omega_s P_m}\right) e^{\frac{\sigma_s^2}{\Omega_s P_m}}.$$

D. Scenario 4

Consider the final scenario, where the SU-Tx has knowledge of only the mean values of g_p and g_{sp}. Here we have

$$\Pr\left(\frac{P_p g_p}{P_s g_{sp} + \sigma_p^2} \geq \gamma_T \bigg| \Omega_p, \Omega_{sp}\right) = 1 - \alpha. \quad (30)$$

Using conditioning (30) can be given as

$$\mathbb{E}\left[\Pr\left(P_p g_p \geq \gamma_T\left(P_s g_{sp} + \sigma_p^2\right) \bigg| g_{sp}\right)\right] = 1 - \alpha, \quad (31)$$

which after some manipulation gives the transmit power P_s,

$$P_s = \frac{P_p \Omega_p}{\gamma_T \Omega_{sp}}\left(\frac{e^{-\frac{\gamma_T \sigma_p^2}{P_p \Omega_p}}}{1 - \alpha} - 1\right). \quad (32)$$

Here, P_s and thus P_t, are deterministic, depending simply on the system parameters. The latter is given by

$$P_t = \begin{cases} 0 & P_s < 0 \\ P_s & 0 < P_s < P_m \\ P_m & P_s > P_m \end{cases}. \quad (33)$$

Similarly to *Scenario 3*, from (32) and (33) we conclude that SU transmission occurs when (24) is satisfied.

The cdf of γ is given by

$$F_\gamma(x) = 1 - e^{-\frac{x}{P_t \Omega_s}}, \quad (34)$$

and, from (4), the capacity cdf is

$$F_C(y) = 1 - e^{-\frac{\sigma_p^2(2^y - 1)}{P_t \Omega_s}}. \quad (35)$$

The pdf of C can be trivially obtained by differentiating (35). Furthermore, we can evaluate the mean SU capacity using (5),

$$\bar{C} = \frac{1}{\ln(2)P_t \Omega_s} \int_0^\infty \ln\left(1 + \frac{x}{\sigma_s^2}\right) e^{-\frac{x}{P_t \Omega_s}} dx \quad (36)$$

$$= \frac{1}{\ln(2)} e^{\frac{\sigma_s^2}{P_t \Omega_s}} E_1\left(\frac{\sigma_s^2}{P_t \Omega_s}\right)$$

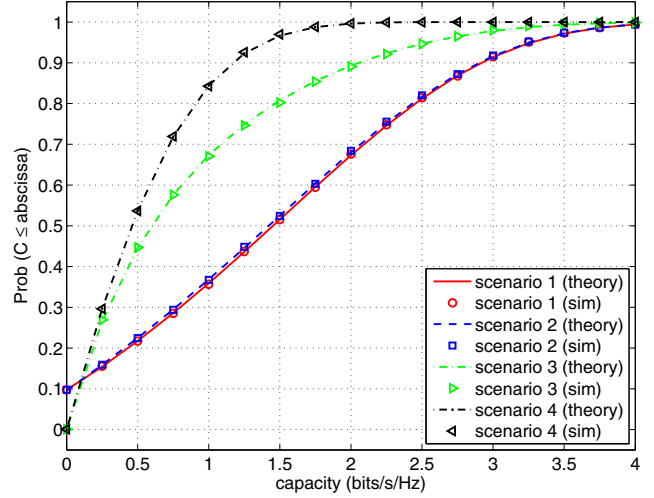where $E_1(x) = \int_x^\infty e^{-t}/t \, dt$ denotes the exponential integral function of order one.

Fig. 2. SU capacity cdf for *Scenarios* 1-4 ($c_1 = c_2 = 0.1$).

IV. SIMULATION RESULTS AND DISCUSSION

We now present simulation results used to validate the analytical expressions derived in Section III, and to compare capacity values achievable under each scenario. In all simulations we have set $P_p/\sigma_p^2 = P_m/\sigma_s^2 = 0$ dB and $\Omega_p/\sigma_p^2 = \Omega_s/\sigma_s^2 = 5$ dB, where we assume $\sigma_p^2 = \sigma_s^2$. In *Scenarios* 2-4 we set $\alpha = 0.1$. The remaining system variables were parameterized using

$$c_1 = \frac{\Omega_{sp}}{\Omega_s}, \quad (37)$$

that is, the ratio of interference to desired channel strength, and

$$c_2 = \frac{\gamma_T}{P_p \Omega_p/\sigma_p^2}, \quad (38)$$

which is the ratio of the minimum target SINR to the actual SNR at the PU-Rx. Hence, increasing c_2 corresponds to reducing the allowable interference, with the case of $c_2 = 1$ corresponding to zero allowable interference. The parameter values used in the simulations are indicated in the relevant figures.

Figures 2 and 3 show the SU capacity cdf for all scenarios, with lines corresponding to analytical expressions using (10), (18), (27) and (35), and symbols indicating simulatated results. From these results we observe that *Scenarios* 1 and 2 result in similar performance, even in the case of $c_1 = 0.9$, that is where the SU interference is very prominent, thus making it difficult to satisfy the PU SINR constraint (1). Furthermore, lack of knowledge of the PU-PU (that is, knowing only the mean Ω_p) greatly reduces the achievable capacity of the SU. From the point of view of the SU-PU collaboration this is a positive result, since g_p is likely to be known at the PU-Rx. Hence, the PU already has the most useful information and does not need to estimate g_{sp}, which might make the price of co-existence too high from the PU perspective.

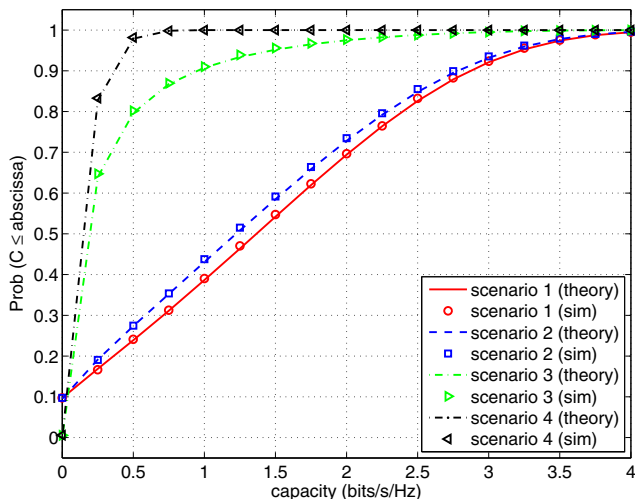In comparing all four scenarios in Figs. 2 and 3, we used $c_2 = 0.1$, which is very generous towards the SU.

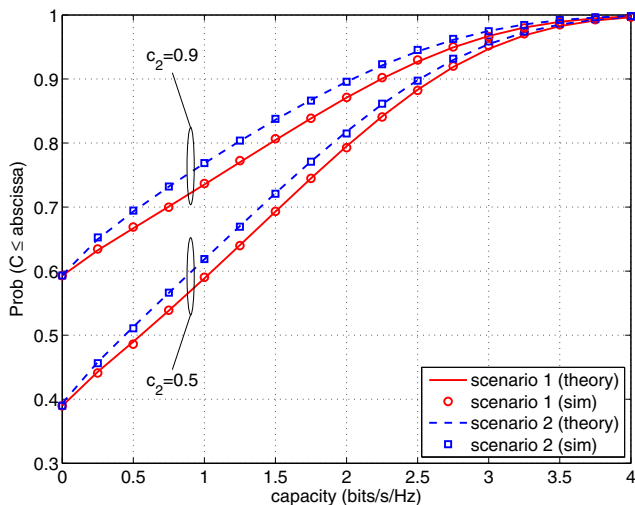Fig. 3. SU capacity cdf for *Scenarios* 1-4 ($c_1 = 0.9, c_2 = 0.1$).

Fig. 4. SU capacity cdf for *Scenarios* 1 and 2 ($c_1 = 0.1$; $c_2 = 0.5$, $c_2 = 0.9$)

Fig. 5. SU capacity cdf for *Scenarios* 1 and 2 for varying c_1 ($c_2 = 0.5$, $c_2 = 0.9$).

capacity cdf under the constraint of a minimum SINR at the PU-Rx, which were verified by means of Monte Carlo simulations. The results demonstrate the importance of the PU-PU CSI, which was shown to be much greater than that of the SU-Tx to PU-Rx link. More importantly, we show that for reasonable system parameters SU transmission does not occur unless the PU-PU CSI is known.

REFERENCES

[1] J. Mitola III, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. dissertation, KTH Royal Institute of Technology, Sweden, May 2000.

[2] T. A. Weiss and F. K. Jondral, "Spectrum pooling: An innovative stratergy for the enhancement of spectrum efficiency," *IEEE Commun. Mag.*, vol. 42, pp. 8–14, March 2004.

[3] S. A. Jafar and S. Srinivasa, "Capacity limits of cognitive radio with distributed and dynamic spectral activity," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 529–537, April 2007.

[4] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 649–658, February 2007.

[5] L. Musavian and S. Aissa, "Fundamental capacity limits of spectrum-sharing channels with imperfect feedback," in *Proc. IEEE GLOBECOM 2007*, November 2007, pp. 1385–1389.

[6] H. A. Suraweera, J. Gao, P. J. Smith, M. Shafi, and M. Faulkner, "Channel capacity limits of cognitive radio in asymmetric fading environments," in *Proc. IEEE ICC 2008*, May 2008, pp. 4048–4053.

[7] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, accepted, January 2010.

[8] R. Zhang, "Optimal power control over fading cognitive radio channels by exploiting primary user CSI," in *Proc. IEEE GLOBECOM 2008*, November 2008, pp. 1–5.

[9] X. Kang, Y.-C. Liang, A. Nallanathan, H. K. Garg, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 940–950, February 2009.

[10] C.-X. Wang, X. Hong, H.-H. Chen, and J. Thompson, "On capacity of cognitive radio networks with average interference power constraints," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 1620–1625, April 2009.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed. San Diego, CA: Academic Press, 2007.

Recalling (24), and noting that using (38) it can be written as $\alpha > 1 - e^{-c_2}$, we see that SU transmission in *Scenarios* 3 and 4 occurs only for large values of α or for small values of c_2. That is, without the knowledge of g_p, the SU can only operate if the PU is willing to accept large amounts of interference. Figures 4 presents the capacity results for *Scenarios* 1 and 2 with more realistic values of $c_2 = 0.5$ and $c_2 = 0.9$, where (24) prevents SU transmission under *Scenarios* 3 and 4. Finally, Fig. 5 shows the probability $\Pr(C \leq 1)$, where capacity is measured in bps/Hz, as a function of c_1. As expected, for a constant c_2, the performance under *Scenario* 2 diverges from the baseline *Scenario* 1 with increasing c_1, that is as the amount of interference to the PU increases.

V. CONCLUSIONS

In this paper, we have examined the effects of channel knowledge availability on the capacity of the SU. Considering four scenarios, we derived an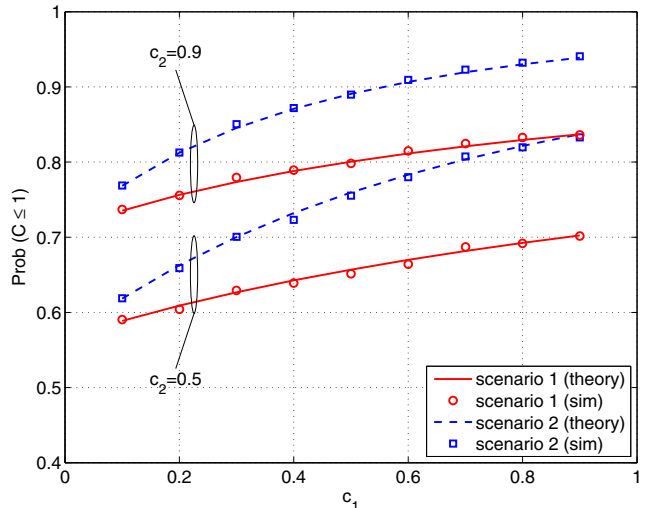alytical expressions for the SU